UWO-TH-05/08
hep-th/0509083

Transport properties of cascading gauge theories

Alex Buchel

Perimeter Institute for Theoretical Physics
Waterloo, Ontario N2J 2W9, Canada

Department of Applied Mathematics, University of Western Ontario
London, Ontario N6A 5B7, Canada

Abstract

Cascading gauge theories of Klebanov *et.al.* provide a model within a framework of gauge theory/string theory duality for a four dimensional non-conformal gauge theory with a spontaneously generated mass scale. Using the dual supergravity description we study sound wave propagation in strongly coupled cascading gauge theory plasma. We analytically compute the speed of sound and the bulk viscosity of cascading gauge theory plasma at a temperature much larger than the strong coupling scale of the theory. The sound wave dispersion relation is obtained from the hydrodynamic pole in the stress-energy tensor two-point correlation function. The speed of sound extracted from the pole of the correlation function agrees with its value computed in [hep-th/0506002] using the equation of state. We find that the bulk viscosity of the hot cascading gauge theory plasma is non-zero at the leading order in the deviation from conformality.

September 2005

1 Introduction

The correspondence between gauge theories and string theory of Maldacena [1, 2] has become a valuable tool in analyzing near-equilibrium dynamics of strongly coupled gauge theory plasma [3–10, 13, 11, 12, 14–17]. The research in this direction is primarily motivated by its potential application for the hydrodynamic description of the QCD quark-gluon plasma believed to be produced in heavy ion collision experiments at RHIC [18–20].

Previously, the dual supergravity computations [8] were shown to reproduce the expected dispersion relation for sound waves in strongly coupled $\mathcal{N}=4$ supersymmetric Yang-Mills (SYM) theory plasma

$$\omega(q) = v_s q - i\ \frac{2q^2}{3T}\ \frac{\eta}{s}\ \left(1+\frac{3\zeta}{4\eta}\right)\,, \tag{1}$$

where v_s, η, s, ζ are the plasma sound speed, shear viscosity, entropy density, and bulk viscosity correspondingly. Conformal symmetry of the $\mathcal{N}=4$ gauge theory insures that

$$v_s = \frac{1}{\sqrt{3}}\,, \qquad \zeta = 0\,. \tag{2}$$

Non-conformal gauge theories, and QCD in particular, is expected to have nonvanishing bulk viscosity. Additionally, the presence of a scale in a gauge theory breaks conformal invariance responsible for speed of sound expression in Eq. (2). To see this note that in conformal theories the energy density and the pressure are related as follows $\epsilon = 3P$, thus, $v_s^2 = \frac{\partial P}{\partial \epsilon} = \frac{1}{3}$.

The first computation for the speed of sound and its attenuation in strongly coupled non-conformal four dimensional gauge theories from the dual supergravity perspective were reported in [17]. Specifically, the model considered there was a mass deformation of the $\mathcal{N}=4$ Yang-Mills theory by giving identical masses m_b to bosonic components of the two SYM chiral multiples and identical masses m_f to fermionic components of the same chiral multiples. Generically $m_f \neq m_b$ and the supersymmetry is completely broken. For a special case $m_f = m_b$ such a deformation leaves eight supersymmetries unbroken, and the model is usually referred to as $\mathcal{N}=2^*$ gauge theory [21–24]. Using finite temperature gauge/gravity correspondence for this mass deformed $\mathcal{N}=4$ SYM [25] the speed of sound and the ratio of the shear to bulk viscosity was found,

respectively,

$$v_s = \frac{1}{\sqrt{3}} \left(1 - \frac{\left[\Gamma\left(\frac{3}{4}\right)\right]^4}{3\pi^4} \left(\frac{m_f}{T}\right)^2 - \frac{1}{18\pi^4} \left(\frac{m_b}{T}\right)^4 + \cdots \right) , \tag{3}$$

$$\frac{\zeta}{\eta} = \beta_f^\Gamma \frac{\left[\Gamma\left(\frac{3}{4}\right)\right]^4}{3\pi^3} \left(\frac{m_f}{T}\right)^2 + \frac{\beta_b^\Gamma}{432\pi^2} \left(\frac{m_b}{T}\right)^4 + \cdots , \tag{4}$$

where $\beta_f^\Gamma \approx 0.9672$, $\beta_b^\Gamma \approx 8.001$, and the ellipses denote higher order terms in m_f/T and m_b/T. From the dependence in Eqs. (3), (4) it follows that at least in the high temperature regime the ratio of bulk viscosity to shear viscosity is proportional to the deviation of the speed of sound squared from its value in conformal theory,

$$\frac{\zeta}{\eta} \simeq -\kappa \left(v_s^2 - \frac{1}{3} \right) , \tag{5}$$

where $\kappa = 3\pi\beta_f^\Gamma/2 \approx 4.558$ for $m_b = 0$, and $\kappa = \pi^2\beta_b^\Gamma/16 \approx 4.935$ for $m_f = 0$.

In this paper we extend analysis of the transport properties in strongly coupled non-conformal gauge theory plasma to theories with spontaneously generated mass scale. More precisely, we analytically compute the speed of sound and its attenuation in 'cascading gauge theories' [26–28] (see [29] for a recent review) at temperature much higher than the deconfinement and the chiral symmetry breaking scales of the theory. The relevant finite temperature deformations of the theory were discussed in [30–32]; the holographic renormalization of the cascading gauge theories was explained in [33]. Moreover, in [33] the equation of state describing hot cascading gauge theory plasma was obtained from which the speed of sound was predicted to be

$$v_s^2 = \frac{\partial \mathcal{P}}{\partial \epsilon} = \frac{\frac{\partial \mathcal{P}}{\partial T}}{\frac{\partial \epsilon}{\partial T}} = \frac{1}{3} - \frac{2}{9 \ln \frac{T}{\Lambda}} + \cdots , \tag{6}$$

where Λ is the strong coupling scale of the theory and ellipses denote subdominant terms for $T \gg \Lambda$. Here, we extract the dispersion relation (1) for the cascading gauge theory plasma from the pole of the thermal two-point function of certain components of the stress-energy tensor in the hydrodynamic approximation, *i.e.*, in the regime where energy and momentum are small in comparison with the inverse thermal wavelength ($\omega/T \ll 1, q/T \ll 1$), and at high temperature $T \gg \Lambda$. The latter computation is equivalent [16] to determining the dispersion for the lowest quasinormal mode in the corresponding black brane geometry (in our case [32]). The general prescription for

computing the quasinormal modes introduced in [16] has been applied recently to a variety of gauge/gravity duality examples [34, 17, 35].

As the computations are rather technical, we begin by summarizing our results in the next section. In section 3 we review five-dimensional effective supergravity description dual to chirally symmetric phase of the cascading gauge theory [33] and the explicit analytic construction of the background black brane geometry [32] dual to thermal cascading gauge theory at temperature much larger than its strong coupling scale. In section 4 we study fluctuations of the corresponding black brane geometry dual to a sound wave mode of the cascading gauge theory plasma. We introduce gauge invariant fluctuations and obtain their equations of motion. These equations of motion are valid beyond the hydrodynamic approximation, and for arbitrary temperature (as long as it is higher than the chiral symmetry breaking scale of the cascading gauge theory). In section 5 we derive and solve fluctuation equations in the hydrodynamic limit and at temperatures much higher than the cascading gauge theory strong coupling scale. Imposing Dirichlet condition on the gauge invariant fluctuations at the boundary of the background black brane geometry determines [16] the dispersion relation for the lowest quasinormal frequency (1). Using the universality of the shear viscosity to entropy density ratio in strongly coupled gauge theory [11,12,15], computed dispersion relation can further be used to evaluate the ratio of bulk to shear viscosities. Some computational details are delegated to Appendices A, B and C.

2 Summary of results

Cascading gauge theory at a given high-energy scale resembles $\mathcal{N}=1$ supersymmetric $SU(K_*)\times SU(K_*+P)$ gauge theory with two bifundamental and two anti-fundamental chiral superfields and certain superpotential, which is quartic in superfields. Unlike ordinary quiver gauge theories, an 'effective rank' of cascading gauge theories depends on an energy scale at which the theory is probed [30, 32, 33]

$$K_* \equiv K_*(E) \sim 2P^2 \ \ln\frac{E}{\Lambda}\,, \qquad E \gg \Lambda\,. \tag{7}$$

At a given temperature T cascading gauge theory is probed at energy scale $E\sim T$, and as $T\gg\Lambda$, $K_*(T)\gg P^2$. In this regime the thermal properties of the theory [32, 33] are very similar to those of the $\mathcal{N}=1$ $SU(K_*)\times SU(K_*)$ superconformal gauge theory

of Klebanov and Witten [39], with

$$\delta_{cascade} \equiv \frac{P^2}{K_*} \tag{8}$$

being the deformation parameter. Clearly, at the temperature increases, $\delta_{cascade}$ becomes smaller and smaller. From the equation of state for the cascading gauge theory one obtains the speed of sound in cascading gauge theory plasma as [33]

$$v_s^2 = \frac{1}{3} - \frac{4}{9}\ \delta_{cascade} + \mathcal{O}(\delta_{cascade}^2)\,. \tag{9}$$

In this paper we compute the dispertion relation for the lowest quasinormal mode in the black brane geometry holographically dual to thermal cascading gauge theory. At high temperature, $\delta_{cascade} \ll 1$, we find (see Eqs. (71), (76), (94), (110))

$$\omega(q) = \frac{1}{\sqrt{3}}\left(1 - \frac{2}{3}\ \delta_{cascade}\right)q - i\ \frac{q^2}{6\pi T}\left(1 + \frac{2}{3}\delta_{cascade}\right) + \mathcal{O}\left(\frac{q^3}{T^2}, \delta_{cascade}^2\right)\,, \tag{10}$$

from which we precisely reproduce the speed of sound extracted from the equation of state (9). Furthermore, comparing Eqs. (1) and (10) and using the universal result [11, 12, 15]

$$\frac{\eta}{s} = \frac{1}{4\pi}\,, \tag{11}$$

we arrive at the 'phenomenological relation' for cascading gauge theory plasma similar to (5)

$$\frac{\zeta}{\eta} = -2\ \left(v_s^2 - \frac{1}{3}\right) + \mathcal{O}\left(\delta_{cascade}^2\right)\,. \tag{12}$$

Most importantly, it appears that phenomenological relation

$$\frac{\zeta}{\eta} \sim -\mathcal{O}(1) \times\ \left(v_s^2 - \frac{1}{3}\right) + \mathcal{O}\left(\left[v_s^2 - \frac{1}{3}\right]^2\right) \tag{13}$$

is a robust prediction for hot strongly coupled non-conformal gauge theory plasma no matter whether scale invariance is broken explicitly (by masses as in Eq. (5)) or spontaneously (by a strong coupling scale as in Eq. (12)). As such, we expect it to be of relevance to real QCD quark-gluon plasma. (Note that the result (13) appears to disagree with the estimates $\zeta \sim \eta\,(v_s^2 - 1/3)^2$ [36, 37], later criticized in [38].)

3 Effective actions and equations of motion

Effective 5d action describing supergravity dual to cascading gauge theories is given by [33]

$$
\begin{split}
S_5 = \frac{1}{16\pi G_5} \int_{\mathcal{M}_5} \hat{\text{vol}}_{\mathcal{M}_5} \biggl\{ & \Omega_1 \Omega_2^4 \biggl(R_{10} - \frac{1}{2} \nabla_\mu \Phi \nabla^\mu \Phi \biggr) - P^2 \Omega_1 e^{-\Phi} \biggl(\frac{\nabla_\mu K \nabla^\mu K}{4P^4} + \frac{e^{2\Phi}}{\Omega_1^2} \biggr) \\
& - \frac{1}{2} \frac{K^2}{\Omega_1 \Omega_2^4} \biggr\} ,
\end{split}
\tag{14}
$$

where R_{10} is given by

$$
\begin{split}
R_{10} = & \hat{R}_5 - 2\Omega_1^{-1} \hat{g}^{\lambda\nu} \Bigl(\nabla_\lambda \nabla_\nu \Omega_1 \Bigr) - 8\Omega_2^{-1} \hat{g}^{\lambda\nu} \Bigl(\nabla_\lambda \nabla_\nu \Omega_2 \Bigr) \\
& - 4\hat{g}^{\lambda\nu} \Bigl(2\ \Omega_1^{-1} \Omega_2^{-1}\ \nabla_\lambda \Omega_1 \nabla_\nu \Omega_2 + 3\ \Omega_2^{-2}\ \nabla_\lambda \Omega_2 \nabla_\nu \Omega_2 \Bigr) \\
& + 24\ \Omega_2^{-2} - 4\ \Omega_1^2\ \Omega_2^{-4} ,
\end{split}
\tag{15}
$$

with $\hat{R}_5$ being the five dimensional Ricci scalar of the metric

$$
d\hat{s}_5^2 = \hat{g}_{\mu\nu}(y) dy^\mu dy^\nu ,
\tag{16}
$$

and G_5 is the five dimensional effective gravitational constant

$$
G_5 \equiv \frac{G_{10}}{\text{vol}_{T^{1,1}}} .
\tag{17}
$$

We find it convenient to rewrite the action (14) in 5d Einstein frame. The latter is achieved with the following rescaling

$$
\hat{g}_{\mu\nu} \longrightarrow g_{\mu\nu} \equiv \Omega_1^{2/3} \Omega_2^{8/3}\ \hat{g}_{\mu\nu} .
\tag{18}
$$

Further introducing

$$
\Omega_1 = e^{f-4w} , \qquad \Omega_2 = e^{f+w} ,
\tag{19}
$$

the five dimensional effective action becomes

$$
\begin{split}
S_5 = \frac{1}{16\pi G_5} \int_{\mathcal{M}_5} \text{vol}_{\mathcal{M}_5} \biggl\{ & R_5 - \frac{40}{3} (\partial f)^2 - 20 (\partial w)^2 - \frac{1}{2} (\partial \Phi)^2 - \frac{1}{4P^2} (\partial K)^2 e^{-\Phi - 4f - 4w} \\
& - \mathcal{P} \biggr\} ,
\end{split}
\tag{20}
$$

where we defined

$$\mathcal{P} = -24e^{-\frac{16}{3}f-2w} + 4e^{-\frac{16}{3}f-12w} + P^2 e^{\Phi-\frac{28}{3}f+4w} + \frac{1}{2}K^2 e^{-\frac{40}{3}f} . \tag{21}$$

From Eq. (20) we obtain the following equations of motion

$$0 = \Box f + \frac{3}{80P^2}e^{-\Phi-4f-4w}(\partial K)^2 - \frac{3}{80}\frac{\partial\mathcal{P}}{\partial f} , \tag{22}$$

$$0 = \Box w + \frac{1}{40P^2}e^{-\Phi-4f-4w}(\partial K)^2 - \frac{1}{40}\frac{\partial\mathcal{P}}{\partial w} , \tag{23}$$

$$0 = \Box \Phi + \frac{1}{4P^2}e^{-\Phi-4f-4w}(\partial K)^2 - \frac{\partial\mathcal{P}}{\partial \Phi} , \tag{24}$$

$$0 = \Box K - \partial K\partial(\Phi + 4f + 4w) - 2P^2 e^{\Phi+4f+4w}\frac{\partial\mathcal{P}}{\partial K} \tag{25}$$

$$R_{5\mu\nu} = \frac{40}{3}\ \partial_\mu f\partial_\nu f + 20\ \partial_\mu w\partial_\nu w + \frac{1}{2}\ \partial_\mu \Phi\partial_\nu \Phi + \frac{1}{4P^2}e^{-\Phi-4f-4w}\ \partial_\mu K\partial_\nu K + \frac{1}{3}g_{\mu\nu}\ \mathcal{P} . \tag{26}$$

3.1 Black brane background geometry

Taking the black brane metric ansatz

$$ds_5^2 = -c_1^2\ dt^2 + c_2^2\ d\vec{x}^2 + c_3^2\ dr^2 , \tag{27}$$

and assuming that all matter fields $\{f, w, \Phi, K\}$ depend on the radial coordinate r only, the background equations of motion are

$$0 = f'' + f'\ \left[\ln\frac{c_1 c_2^3}{c_3}\right]' + \frac{3}{80P^2}e^{-\Phi-4f-4w}(K')^2 - \frac{3}{80}\ c_3^2\frac{\partial\mathcal{P}}{\partial f} , \tag{28}$$

$$0 = w'' + w'\ \left[\ln\frac{c_1 c_2^3}{c_3}\right]' + \frac{1}{40P^2}e^{-\Phi-4f-4w}(K')^2 - \frac{1}{40}\ c_3^2\frac{\partial\mathcal{P}}{\partial w} , \tag{29}$$

$$0 = \Phi'' + \Phi' \left[\ln \frac{c_1 c_2^3}{c_3}\right]' + \frac{1}{4P^2} e^{-\Phi-4f-4w} (K')^2 - c_3^2 \frac{\partial \mathcal{P}}{\partial \Phi}\,, \tag{30}$$

$$0 = K'' + K' \left[\ln \frac{c_1 c_2^3}{c_3}\right]' - K'[\Phi + 4f + 4w]' - 2P^2 c_3^2 e^{\Phi+4f+4w} \frac{\partial \mathcal{P}}{\partial K}\,, \tag{31}$$

$$0 = c_1'' + c_1' \left[\ln \frac{c_2^3}{c_3}\right]' + \frac{1}{3} c_1 c_3^2 \mathcal{P}\,, \tag{32}$$

$$0 = c_2'' + c_2' \left[\ln \frac{c_1 c_2^2}{c_3}\right]' + \frac{1}{3} c_2 c_3^2 \mathcal{P}\,. \tag{33}$$

Additionally, there is a first order constraint

$$0 = (\Phi')^2 + \frac{80}{3}(f')^2 + 40(w')^2 + \frac{1}{2P^2} e^{-\Phi-4f-4w} (K')^2 - 12[\ln\ c_2]'[\ln\ c_1 c_2]' - 2c_3^2 \mathcal{P}\,. \tag{34}$$

Notice that Eqs. (28)-(34) are equivalent to the set of equations derived in [31, 32] provided we identify

$$c_1 \equiv e^{\frac{5}{3}f + \hat{z} - 3\hat{x}}\,, \qquad c_2 \equiv e^{\frac{5}{3}f + \hat{z} + \hat{x}}\,, \qquad c_3 \equiv e^{\frac{5}{3}f - \hat{z} + 5\hat{y}}\,, \qquad f \equiv \hat{y} - \hat{z}\,, \tag{35}$$

where $\{\hat{x}, \hat{y}, \hat{z}\}$ are correspondingly $\{x, y, z\}$ of [31, 32]. We would need solution to Eqs. (28)-(34) to leading order in P^2. This was originally done in [32], but we repeat the main steps to set up conventions. We find convenient to use a new radial coordinate

$$x \equiv \frac{c_1}{c_2}\,. \tag{36}$$

In terms of x Eqs. (28)-(34) become

$$0 = c_2'' - \frac{5}{c_2}(c_2')^2 - \frac{1}{x} c_2' + c_2 \left\{ \frac{40}{9}(f')^2 + \frac{20}{3}(w')^2 + \frac{1}{6}(\Phi')^2 + \frac{1}{12P^2} e^{-\Phi-4f-4w} (K')^2 \right\}, \tag{37}$$

$$\begin{split} 0 = f'' + \frac{1}{x} f' + \frac{3}{80P^2} e^{-\Phi-4f-4w} (K')^2 + \frac{\partial \ln \mathcal{P}}{\partial f} \left\{ \frac{9}{20}([\ln c_2]')^2 + \frac{9}{40x}[\ln c_2]' - \frac{3}{4}(w')^2 \right. \\ \left. - \frac{3}{160}(\Phi')^2 - \frac{1}{2}(f')^2 - \frac{3}{320P^2} e^{-\Phi-4f-4w} (K')^2 \right\}, \end{split} \tag{38}$$

$$
\begin{aligned}
0 =& w'' + \frac{1}{x}w' + \frac{1}{40P^2}e^{-\Phi-4f-4w}(K')^2 + \frac{\partial \ln \mathcal{P}}{\partial w}\ \left\{\frac{3}{10}([\ln c_2]')^2 + \frac{3}{20x}[\ln c_2]' - \frac{1}{2}(w')^2\right. \\
& \left. - \frac{1}{80}(\Phi')^2 - \frac{1}{3}(f')^2 - \frac{1}{160P^2}e^{-\Phi-4f-4w}(K')^2\right\},
\end{aligned}
\tag{39}
$$

$$
\begin{aligned}
0 =& \Phi'' + \frac{1}{x}\Phi' + \frac{1}{4P^2}e^{-\Phi-4f-4w}(K')^2 + \frac{\partial \ln \mathcal{P}}{\partial \Phi}\ \left\{12([\ln c_2]')^2 + \frac{6}{x}[\ln c_2]' - 20(w')^2\right. \\
& \left. - \frac{1}{2}(\Phi')^2 - \frac{40}{3}(f')^2 - \frac{1}{4P^2}e^{-\Phi-4f-4w}(K')^2\right\},
\end{aligned}
\tag{40}
$$

$$
\begin{aligned}
0 =& K'' + [\ln x - \Phi - 4f - 4w]'K' + \frac{\partial \ln \mathcal{P}}{\partial \Phi}\ P^2 e^{\Phi+4f+4w}\left\{24([\ln c_2]')^2 + \frac{12}{x}[\ln c_2]'\right. \\
& \left. - 40(w')^2 - (\Phi')^2 - \frac{80}{3}(f')^2 - \frac{1}{2P^2}e^{-\Phi-4f-4w}(K')^2\right\}.
\end{aligned}
\tag{41}
$$

Notice that Eqs. (37)-(41) have an exact scaling symmetry

$$
c_2 \to \lambda\ c_2\,, \qquad f \to f\,, \qquad w \to w\,, \qquad \Phi \to \Phi\,, \qquad K \to K\,, \tag{42}
$$

for a constant λ. We will see later that this symmetry has an extension for the fluctuations as well. Physically, this symmetry corresponds to choosing a reference energy scale.

To order $\mathcal{O}(P^2)$ the solution to Eqs. (37)-(41) takes form [32]

$$
\begin{aligned}
& c_2 = \frac{a}{(1-x^2)^{1/4}}\ \left(1 + \frac{P^2}{K_*}\xi(x)\right)\,, \qquad f = -\frac{1}{4}\ln\frac{4}{K_*} + \frac{P^2}{K_*}\eta(x)\,, \\
& w = \frac{P^2}{K_*}\psi(x)\,, \qquad \Phi = \frac{P^2}{K_*}\zeta(x), \qquad K = K_* + P^2\kappa(x)\,,
\end{aligned}
\tag{43}
$$

where a is a constant nonextremality parameter, and

$$
\begin{aligned}
& \xi = \frac{1}{12}(1 - \ln(1-x^2))\,, \qquad \kappa = -\frac{1}{2}\ln(1-x^2)\,, \\
& \zeta = \frac{K_*}{P^2}\Phi_{horizon} + \frac{\pi^2}{12} - \frac{1}{2}\mathrm{dilog(x)} + \frac{1}{2}\mathrm{dilog(1+x)} - \frac{1}{2}\ln \mathrm{x} \ln(1-\mathrm{x})\,, \\
& \eta = -\frac{3(1+x^2)}{80(1-x^2)}\left(\mathrm{dilog(1-x^2)} - \frac{\pi^2}{6}\right) + \frac{1}{20}(1-\ln(1-x^2))\,.
\end{aligned}
\tag{44}
$$

Furthermore, ψ satisfies the linear differential equation

$$0 = \psi'' + \frac{1}{x}\psi' - \frac{3}{(1-x^2)^2}\psi - \frac{1}{10(1-x^2)}\,, \tag{45}$$

with the boundary condition

$$\psi = \psi_{horizon} + \mathcal{O}(x^2)\,, \qquad \psi = -\frac{1}{30}(1-x^2) + \mathcal{O}\Big((1-x^2)^{3/2}\Big)\,, \tag{46}$$

where the second boundary condition will uniquely determine $\psi_{horizon}$.

Finally, the (exact in P^2) temperature of the nonextremal solution is given by

$$(2\pi T)^2 = -\frac{\mathcal{P}c_2^3}{6c_2''}\Bigg|_{x\to 0_+}\,. \tag{47}$$

4 Fluctuations

Now we study fluctuations in the background geometry

$$\begin{aligned} g_{\mu\nu} &\to g_{\mu\nu} + h_{\mu\nu}\,, \\ f &\to f + \delta f\,, \\ w &\to w + \delta w\,, \\ \Phi &\to \Phi + \delta\Phi\,, \\ K &\to K + \delta K\,, \end{aligned} \tag{48}$$

where $\{g_{\mu\nu}, f, w, \Phi, K\}$ are the black brane background configuration (satisfying Eqs. (28)-(34)), and $\{h_{\mu\nu}, \delta f, \delta w, \delta\Phi, \delta K\}$ are the fluctuations. We choose the gauge

$$h_{tr} = h_{x_ir} = h_{rr} = 0\,. \tag{49}$$

Additionally, we assume that all the fluctuations depend only on (t, x_3, r), *i.e.*, we have an $O(2)$ rotational symmetry in the x_1x_2 plane.

At a linearized level we find that the following sets of fluctuations decouple from each other

$$\begin{aligned} &\{h_{x_1x_2}\}\,, \\ &\{h_{x_1x_1} - h_{x_2x_2}\}\,, \\ &\{h_{tx_1},\ h_{x_1x_3}\}\,, \\ &\{h_{tx_2},\ h_{x_2x_3}\}\,, \\ &\{h_{tt},\ h_{aa} \equiv h_{x_1x_1} + h_{x_2x_2},\ h_{tx_3},\ h_{x_3x_3},\ \delta f,\ \delta w,\ \delta\Phi,\ \delta K\}\,. \end{aligned} \tag{50}$$

The last set of fluctuations is a holographic dual to the sound waves in cascading gauge theory plasma which is of interest here. Introduce

$$
\begin{split}
h_{tt} =& c_1^2\ \hat{h}_{tt} = e^{-i\omega t+iqx_3}\ c_1^2\ H_{tt}\,,\\
h_{tz} =& c_2^2\ \hat{h}_{tz} = e^{-i\omega t+iqx_3}\ c_2^2\ H_{tz}\,,\\
h_{aa} =& c_2^2\ \hat{h}_{aa} = e^{-i\omega t+iqx_3}\ c_2^2\ H_{aa}\,,\\
h_{zz} =& c_2^2\ \hat{h}_{zz} = e^{-i\omega t+iqx_3}\ c_2^2\ H_{zz}\,,\\
\delta f =& e^{-i\omega t+iqx_3}\ \mathcal{F}\,,\\
\delta w =& e^{-i\omega t+iqx_3}\ \Omega\,,\\
\delta \Phi =& e^{-i\omega t+iqx_3}\ p\,,\\
\delta K =& e^{-i\omega t+iqx_3}\ \mathcal{K}\,,\\
\hat{h}_{ii} =& \hat{h}_{aa}+\hat{h}_{zz}\,,\qquad H_{ii}=H_{aa}+H_{zz}\,,
\end{split}
\tag{51}
$$

where $\{H_{tt},H_{tz},H_{aa},H_{zz},\mathcal{F},\Omega,p,\mathcal{K}\}$ are functions of a radial coordinate only. Expanding at a linearized level Eqs. (22)-(26) with Eq. (48) and Eq. (51) we find the following coupled system of ODE's

$$
\begin{split}
0 =& H_{tt}''+H_{tt}'\left[\ln\frac{c_1^2c_2^3}{c_3}\right]'-H_{ii}'\left[\ln c_1\right]'-\frac{c_3^2}{c_1^2}\left(q^2\frac{c_1^2}{c_2^2}\ H_{tt}+\omega^2\ H_{ii}+2\omega q\ H_{tz}\right)\\
&-\frac23 c_3^2\left(\frac{\partial\mathcal{P}}{\partial f}\ \mathcal{F}+\frac{\partial\mathcal{P}}{\partial w}\ \Omega+\frac{\partial\mathcal{P}}{\partial \Phi}\ p+\frac{\partial\mathcal{P}}{\partial K}\ \mathcal{K}\right)\,,
\end{split}
\tag{52}
$$

$$
0 = H_{tz}''+H_{tz}'\ \left[\ln\frac{c_2^5}{c_1c_3}\right]'+\frac{c_3^2}{c_2^2}\ \omega q\ H_{aa}\,,
\tag{53}
$$

$$
\begin{split}
0 =& H_{aa}''+H_{aa}'\ \left[\ln\frac{c_1c_2^5}{c_3}\right]'+(H_{zz}'-H_{tt}')\ [\ln c_2^2]'+\frac{c_3^2}{c_1^2}\left(\omega^2-q^2\frac{c_1^2}{c_2^2}\right)\ H_{aa}\\
&+\frac43 c_3^2\left(\frac{\partial\mathcal{P}}{\partial f}\ \mathcal{F}+\frac{\partial\mathcal{P}}{\partial w}\ \Omega+\frac{\partial\mathcal{P}}{\partial \Phi}\ p+\frac{\partial\mathcal{P}}{\partial K}\ \mathcal{K}\right)\,,
\end{split}
\tag{54}
$$

$$
\begin{split}
0 =& H_{zz}''+H_{zz}'\ \left[\ln\frac{c_1c_2^4}{c_3}\right]'+(H_{aa}'-H_{tt}')\ [\ln c_2]'\\
&+\frac{c_3^2}{c_1^2}\left(\omega^2\ H_{zz}+2\omega q\ H_{tz}+q^2\frac{c_1^2}{c_2^2}(H_{tt}-H_{aa})\right)\\
&+\frac23 c_3^2\left(\frac{\partial\mathcal{P}}{\partial f}\ \mathcal{F}+\frac{\partial\mathcal{P}}{\partial w}\ \Omega+\frac{\partial\mathcal{P}}{\partial \Phi}\ p+\frac{\partial\mathcal{P}}{\partial K}\ \mathcal{K}\right)\,,
\end{split}
\tag{55}
$$

$$
\begin{aligned}
0 =&\mathcal{F}'' + \mathcal{F}' \ \left[\ln \frac{c_1 c_2^3}{c_3}\right]' + \frac{1}{2} f' \ [H_{ii} - H_{tt}]' + \frac{c_3^2}{c_1^2}\left(\omega^2 - q^2 \frac{c_1^2}{c_2^2}\right) \ \mathcal{F} \\
&- \frac{3}{80} c_3^2 \bigg(\frac{\partial^2 \mathcal{P}}{\partial f^2} \ \mathcal{F} + \frac{\partial^2 \mathcal{P}}{\partial f \partial w} \ \Omega + \frac{\partial^2 \mathcal{P}}{\partial f \partial \Phi} \ p + \frac{\partial^2 \mathcal{P}}{\partial f \partial K} \ \mathcal{K} \\
&+ \frac{1}{P^2} \ \frac{(K')^2}{c_3^2} e^{-\Phi - 4f - 4w} - \frac{2}{P^2} \ \frac{K' \mathcal{K}'}{c_3^2} e^{-\Phi - 4f - 4w} (p + 4\mathcal{F} + 4\Omega) \bigg) ,
\end{aligned}
\tag{56}
$$

$$
\begin{aligned}
0 =&\Omega'' + \Omega' \ \left[\ln \frac{c_1 c_2^3}{c_3}\right]' + \frac{1}{2} w' \ [H_{ii} - H_{tt}]' + \frac{c_3^2}{c_1^2}\left(\omega^2 - q^2 \frac{c_1^2}{c_2^2}\right) \ \Omega \\
&- \frac{1}{40} c_3^2 \bigg(\frac{\partial^2 \mathcal{P}}{\partial w \partial f} \ \mathcal{F} + \frac{\partial^2 \mathcal{P}}{\partial w^2} \ \Omega + \frac{\partial^2 \mathcal{P}}{\partial w \partial \Phi} \ p + \frac{\partial^2 \mathcal{P}}{\partial w \partial K} \ \mathcal{K} \\
&+ \frac{1}{P^2} \ \frac{(K')^2}{c_3^2} e^{-\Phi - 4f - 4w} - \frac{2}{P^2} \ \frac{K' \mathcal{K}'}{c_3^2} e^{-\Phi - 4f - 4w} (p + 4\mathcal{F} + 4\Omega) \bigg) ,
\end{aligned}
\tag{57}
$$

$$
\begin{aligned}
0 =&p'' + p' \ \left[\ln \frac{c_1 c_2^3}{c_3}\right]' + \frac{1}{2} \Phi' \ [H_{ii} - H_{tt}]' + \frac{c_3^2}{c_1^2}\left(\omega^2 - q^2 \frac{c_1^2}{c_2^2}\right) \ p \\
&- c_3^2 \bigg(\frac{\partial^2 \mathcal{P}}{\partial \Phi \partial f} \ \mathcal{F} + \frac{\partial^2 \mathcal{P}}{\partial \Phi \partial w} \ \Omega + \frac{\partial^2 \mathcal{P}}{\partial \Phi^2} \ p + \frac{\partial^2 \mathcal{P}}{\partial \Phi \partial K} \ \mathcal{K} \\
&+ \frac{1}{4P^2} \ \frac{(K')^2}{c_3^2} e^{-\Phi - 4f - 4w} (p + 4\mathcal{F} + 4\Omega) - \frac{1}{2P^2} \ \frac{K' \mathcal{K}'}{c_3^2} e^{-\Phi - 4f - 4w} \bigg) ,
\end{aligned}
\tag{58}
$$

$$
\begin{aligned}
0 =&\mathcal{K}'' + \mathcal{K}' \ \left[\ln \frac{c_1 c_2^3}{c_3}\right]' + \frac{1}{2} K' \ [H_{ii} - H_{tt}]' + \frac{c_3^2}{c_1^2}\left(\omega^2 - q^2 \frac{c_1^2}{c_2^2}\right) \ \mathcal{K} \\
&- 2P^2 e^{\Phi + 4f + 4w} c_3^2 \bigg(\frac{\partial^2 \mathcal{P}}{\partial K \partial f} \ \mathcal{F} + \frac{\partial^2 \mathcal{P}}{\partial K \partial w} \ \Omega + \frac{\partial^2 \mathcal{P}}{\partial K \partial \Phi} \ p + \frac{\partial^2 \mathcal{P}}{\partial K^2} \ \mathcal{K} \\
&+ \frac{\partial \mathcal{P}}{\partial K} (p + 4\mathcal{F} + 4\Omega) \bigg) - \mathcal{K}' [\Phi + 4f + 4w]' - K' [p + 4\mathcal{F} + 4\Omega]' ,
\end{aligned}
\tag{59}
$$

where all derivatives $\partial \mathcal{P}$ are evaluated on the background geometry. Additionally, there are three first order constraints associated with the (partially) fixed diffeomorphism invariance

$$
\begin{aligned}
0 =&\omega \left(H_{ii}' + \left[\ln \frac{c_2}{c_1}\right]' \ H_{ii} \right) + q \left(H_{tz}' + 2 \left[\ln \frac{c_2}{c_1}\right]' \ H_{tz} \right) \\
&+ \omega \ \left(\frac{80}{3} f' \mathcal{F} + 40 w' \Omega + \Phi' p + \frac{1}{2P^2} K' \mathcal{K} e^{-\Phi - 4f - 4w} \right) ,
\end{aligned}
\tag{60}
$$

$$
\begin{aligned}
0 =&q \left(H_{tt}' - \left[\ln \frac{c_2}{c_1} \right]' \ H_{tt} \right) + \frac{c_2^2}{c_1^2} \omega \ H_{tz}' - q \ H_{aa} \\
&- q \ \left(\frac{80}{3} f' \mathcal{F} + 40 w' \Omega + \Phi' p + \frac{1}{2P^2} K' \mathcal{K} e^{-\Phi - 4f - 4w} \right) ,
\end{aligned} \tag{61}
$$

$$
\begin{aligned}
0 =&[\ln c_1 c_2^2]' H_{ii}' - [\ln c_2^3]' \ H_{tt}' + \frac{c_3^2}{c_1^2} \left(\omega^2 \ H_{ii} + 2\omega q \ H_{tz} + q^2 \ \frac{c_1^2}{c_2^2} \left(H_{tt} - H_{aa} \right) \right) \\
&+ c_3^2 \left(\frac{\partial \mathcal{P}}{\partial f} \ \mathcal{F} + \frac{\partial \mathcal{P}}{\partial w} \ \Omega + \frac{\partial \mathcal{P}}{\partial \Phi} \ p + \frac{\partial \mathcal{P}}{\partial K} \ \mathcal{K} \right) \\
&- \left(\frac{80}{3} f' \mathcal{F}' + 40 w' \Omega' + \Phi' p' + \frac{1}{2P^2} K' \mathcal{K}' e^{-\Phi - 4f - 4w} \right) \\
&+ \frac{1}{4P^2} (K')^2 e^{-\Phi - 4f - 4w} (p + 4\mathcal{F} + 4\Omega) .
\end{aligned} \tag{62}
$$

We explicitly verified that Eqs. (52)-(59) are consistent with constraints (60)-(62).

Introducing the gauge invariant fluctuations

$$
\begin{aligned}
Z_H =&4 \frac{q}{\omega} \ H_{tz} + 2 \ H_{zz} - H_{aa} \left(1 - \frac{q^2}{\omega^2} \frac{c_1' c_1}{c_2' c_2} \right) + 2 \frac{q^2}{\omega^2} \frac{c_1^2}{c_2^2} \ H_{tt} , \\
Z_f =&\mathcal{F} - \frac{f'}{[\ln c_2^4]'} \ H_{aa} , \\
Z_w =&\Omega - \frac{w'}{[\ln c_2^4]'} \ H_{aa} , \\
Z_\Phi =&p - \frac{\Phi'}{[\ln c_2^4]'} \ H_{aa} , \\
Z_K =&\mathcal{K} - \frac{K'}{[\ln c_2^4]'} \ H_{aa} ,
\end{aligned} \tag{63}
$$

we find from Eqs. (52)-(59), (60)-(62), decoupled [1] set of equations of motion for Z's

$$
0 = A_H Z_H'' + B_H Z_H' + C_H Z_H + D_H Z_f + E_H Z_w + F_H Z_\Phi + G_H Z_K , \tag{64}
$$

$$
0 = A_f Z_f'' + B_f Z_f' + C_f Z_H' + D_f Z_H + E_f Z_f + F_f Z_w + G_f Z_\Phi + H_f Z_K' + I_f Z_K , \tag{65}
$$

[1]To achieve the decoupling one has to use the background equations of motion (28)-(34), *i.e.*, the decoupling occurs only on-shell.

$$
0 = A_w Z_w'' + B_w Z_w' + C_w Z_H' + D_w Z_H + E_w Z_f + F_w Z_w + G_w Z_\Phi + H_w Z_K' + I_w Z_K \,, \tag{66}
$$

$$
0 = A_\Phi Z_\Phi'' + B_\Phi Z_\Phi' + C_\Phi Z_H' + D_\Phi Z_H + E_\Phi Z_f + F_\Phi Z_w + G_\Phi Z_\Phi + H_\Phi Z_K' + I_\Phi Z_K \,, \tag{67}
$$

$$
\begin{split}
0 =& A_K Z_K'' + B_K Z_K' + C_K Z_H' + D_K Z_H + E_K Z_f' + F_K Z_f + G_K Z_w' + H_K Z_w \\
&+ I_K Z_\Phi' + J_K Z_\Phi + K_K Z_K \,,
\end{split} \tag{68}
$$

where we collected connection coefficients $\{A_{...}, \cdots, K_K\}$ in Appendix A. Notice that Eqs. (64)-(68) have an exact scaling symmetry (42) provided the latter is supplemented with

$$
\omega \to \lambda\, \omega \,, \qquad q \to \lambda\, q \,, \tag{69}
$$

while keeping $\{Z_H, Z_f, Z_w, Z_\Phi, Z_K\}$ invariant.

5 Hydrodynamic limit, boundary conditions and small P^2 expansion

We study now physical fluctuation equations (64)-(68) in the hydrodynamics approximation, $\omega \to 0,\ q \to 0$ with $\frac{\omega}{q}$ kept constant, and to leading order in P^2. Similar to the $\mathcal{N} = 2^*$ computations [17], we would need only leading and next-to-leading (in q) solution of (64)-(68). The computations are greatly simplified with judicial choice of the radial coordinate. Choosing the radial coordinate as in (36), we find that at the horizon, $x \to 0_+$, $Z_H \propto x^{\pm i\omega/(2\pi T)}$, and similarly for Z_f, Z_w, Z_Φ, Z_K. The temperature T is given by Eq. (47). Incoming boundary conditions on all physical modes implies that

$$
\begin{split}
&Z_H(x) = x^{-i\mathfrak{w}} z_H(x) \,, \qquad Z_f(x) = x^{-i\mathfrak{w}} z_f(x) \,, \qquad Z_w(x) = x^{-i\mathfrak{w}} z_w(x) \,, \\
&Z_\Phi(x) = x^{-i\mathfrak{w}} z_\Phi(x) \,, \qquad Z_K(x) = x^{-i\mathfrak{w}} z_K(x) \,,
\end{split} \tag{70}
$$

where $\{z_H, z_f, z_w, z_\Phi, z_K\}$ are regular at the horizon; we further introduced

$$
\mathfrak{w} \equiv \frac{\omega}{2\pi T} \,, \qquad \mathfrak{q} \equiv \frac{q}{2\pi T} \,. \tag{71}
$$

There is a single integration constant for these physical modes, namely, the overall scale. Without the loss of generality the latter can be fixed as

$$z_H(x)\Big|_{x\to 0_+} = 1\,. \tag{72}$$

In this case, the pole dispersion relation is simply determined as [16]

$$z_H(x)\Big|_{x\to 1_-} = 0\,. \tag{73}$$

The other boundary conditions (besides regularity at the horizon and (73)) are [16]

$$z_f(x)\Big|_{x\to 1_-} = 0\,,\ \ z_w(x)\Big|_{x\to 1_-} = 0\,,\ \ z_\Phi(x)\Big|_{x\to 1_-} = 0\,,\ \ z_K(x)\Big|_{x\to 1_-} = 0\,. \tag{74}$$

Let's introduce

$$\begin{aligned}
z_H &= \left(z^0_{H,0} + P^2\ z^2_{H,0}\right) + i\ \mathfrak{q}\ \left(z^0_{H,1} + P^2\ z^2_{H,1}\right)\,,\\
z_f &= \left(z^0_{f,0} + P^2\ z^2_{f,0}\right) + i\ \mathfrak{q}\ \left(z^0_{f,1} + P^2\ z^2_{f,1}\right)\,,\\
z_w &= \left(z^0_{w,0} + P^2\ z^2_{w,0}\right) + i\ \mathfrak{q}\ \left(z^0_{w,1} + P^2\ z^2_{w,1}\right)\,,\\
z_\Phi &= \left(z^0_{\Phi,0} + P^2\ z^2_{\Phi,0}\right) + i\ \mathfrak{q}\ \left(z^0_{\Phi,1} + P^2\ z^2_{\Phi,1}\right)\,,\\
z_K &= \left(z^0_{K,0} + P^2\ z^2_{K,0}\right) + i\ \mathfrak{q}\ \left(z^0_{K,1} + P^2\ z^2_{K,1}\right)\,,
\end{aligned} \tag{75}$$

where the lower index refers to either the leading, $\propto \mathfrak{q}^0$, or to the next-to-leading, $\propto \mathfrak{q}^1$, order in the hydrodynamic approximation, and the upper index keeps track of the P^2 deformation parameter. Additionally, as we are interested in the hydrodynamic pole dispersion relation in the stress-energy correlation functions, we find it convenient to parameterize

$$\mathfrak{w} = \frac{\mathfrak{q}}{\sqrt{3}}\left(1+\beta_v\ P^2\right) - i\ \frac{\mathfrak{q}^2}{3}\left(1+\beta_\Gamma\ P^2\right)\,, \tag{76}$$

where the $P^2=0$ coefficients are those of the $\mathcal{N}=4$ plasma, computed in [8], and β_v, β_Γ are constants which are to be determined from the pole dispersion relation (73)

$$z^2_{H,0}\Big|_{x\to 1_-} = 0, \qquad z^2_{H,1}\Big|_{x\to 1_-} = 0\,. \tag{77}$$

Using the high-temperature non-extremal cascading gauge theory flow background (43), parameterizations (75), (76), we obtain from Eqs. (64)-(68)[2] four sets of ODE's describing leading and next-to-leading order in the hydrodynamic approximation and $\mathcal{O}(P^0)$ and $\mathcal{O}(P^2)$ order in the deformation parameter.

In the remaining part of this section each set is discussed in details.

5.1 Equations and solution in $\mathcal{O}(\mathfrak{q}^0)$ and $\mathcal{O}(P^0)$ order

To order $\mathcal{O}(\mathfrak{q}^0)$ and $\mathcal{O}(P^0)$ we find the following set of equations

$$0 =[z_{H,0}^0]'' + \frac{1-3x^2}{x(1+x^2)}\ [z_{H,0}^0]' + \frac{4}{1+x^2}\ z_{H,0}^0 + \frac{32(2(x^3-x)\kappa'+1+x^2)}{(1-x^4)K_*}\ z_{K,0}^0\,, \tag{78}$$

$$0 =[z_{f,0}^0]'' + \frac{1}{x}\ [z_{f,0}^0]' - \frac{8}{(1-x^2)^2}\ z_{f,0}^0 + \frac{3\kappa'}{10K_*}\ [z_{K,0}^0]' + \frac{2}{(1-x^2)^2K_*}\ z_{K,0}^0\,, \tag{79}$$

$$0 =[z_{w,0}^0]'' + \frac{1}{x}\ [z_{w,0}^0]' - \frac{3}{(1-x^2)^2}\ z_{w,0}^0 + \frac{\kappa'}{5K_*}\ [z_{K,0}^0]'\,, \tag{80}$$

$$0 =[z_{\Phi,0}^0]'' + \frac{1}{x}\ [z_{\Phi,0}^0]' + \frac{2\kappa'}{K_*}\ [z_{K,0}^0]'\,, \tag{81}$$

$$0 =[z_{K,0}^0]'' + \frac{1}{x}\ [z_{K,0}^0]'\,. \tag{82}$$

Notice that at this order, the fluctuations couple only through $z_{K,0}^0$, which by itself decouples. The most general solution of Eq. (82) is

$$z_{K,0}^0 = \mathcal{C}_1 + \mathcal{C}_2\ \ln x\,. \tag{83}$$

Regularity at the horizon and Eq. (74) imply that

$$z_{K,0}^0 = 0\,. \tag{84}$$

Vanishing of $z_{K,0}^0 = 0$ decouples all the remaining fluctuations. The pattern that we observe here extends to order $\mathcal{O}(\mathfrak{q}^1)$ in the hydrodynamic approximation and to order

[2]After rewriting them in radial coordinate (36).

$\mathcal{O}(P^2)$ in the deformation parameter:

- the corresponding gauge invariant fluctuations couple only through $z^{\cdots}_{K,\cdots}$;
- $z^{\cdots}_{K,\cdots}$ by itself decouples;
- subject to boundary conditions, the unique solution is $z^{\cdots}_{K,\cdots} = 0$, resulting in decoupling of $\{z^{\cdots}_{H,\cdots}, z^{\cdots}_{f,\cdots}, z^{\cdots}_{w,\cdots}, z^{\cdots}_{\Phi,\cdots}\}$ fluctuations.

Given Eq. (84), the remaining equations can be solved analytically. With boundary conditions (72)-(74) unique solutions are

$$z^0_{H,0} = 1 - x^2 \,, \qquad z^0_{f,0} = z^0_{w,0} = z^0_{\Phi,0} = 0 \,. \tag{85}$$

For $z^0_{H,0}$ we reproduce the sound wave quasinormal mode in the near extremal D3-brane geometry [8, 17] to leading order in the hydrodynamic approximation.

5.2 Equations and solution in $\mathcal{O}(\mathfrak{q}^0)$ and $\mathcal{O}(P^2)$ order

Using Eqs. (84), (85), to order $\mathcal{O}(\mathfrak{q}^0)$ and $\mathcal{O}(P^2)$ we find the following set of equations

$$\begin{split} 0 =&[z^2_{H,0}]'' + \frac{1-3x^2}{x(1+x^2)}\ [z^2_{H,0}]' + \frac{4}{1+x^2}\ z^2_{H,0} + \frac{32(2(x^3-x)\kappa' + 1 + x^2)}{(1-x^4)K_*}\ z^2_{K,0} \\ &+ \frac{8}{3x^2(1+x^2)}\left(\frac{(\kappa')^2}{K_*}(1-x^2)^2(1+x^2) + 6x(1-x^2)^2\xi' + 3x^2\beta_v\right), \end{split} \tag{86}$$

$$\begin{split} 0 =&[z^2_{f,0}]'' + \frac{1}{x}\ [z^2_{f,0}]' - \frac{8}{(1-x^2)^2}\ z^2_{f,0} + \frac{3\kappa'}{10K_*}\ [z^2_{K,0}]' + \frac{2}{(1-x^2)^2K_*}\ z^2_{K,0} \\ &+ \frac{1}{60(x^2-1)x^3K_*}\left(40(x^2-1)\eta' + 40x(4\eta - \kappa) - 7x\right), \end{split} \tag{87}$$

$$\begin{split} 0 =&[z^2_{w,0}]'' + \frac{1}{x}\ [z^2_{w,0}]' - \frac{3}{(1-x^2)^2}\ z^2_{w,0} + \frac{\kappa'}{5K_*}\ [z^2_{K,0}]' \\ &+ \frac{1}{30(x^2-1)x^3K_*}\left(20(x^2-1)\psi' + 30x\psi + x\right), \end{split} \tag{88}$$

$$0 =[z^2_{\Phi,0}]'' + \frac{1}{x}\ [z^2_{\Phi,0}]' + \frac{2\kappa'}{K_*}\ [z^2_{K,0}]' + \frac{1}{3(x^2-1)x^3K_*}\left(2(x^2-1)\zeta' + x\right), \tag{89}$$

$$0 =[z^2_{K,0}]'' + \frac{1}{x}\ [z^2_{K,0}]' + \frac{2}{3(x^2-1)x^3}\left((x^2-1)\kappa' + x\right). \tag{90}$$

Notice that Eqs. (86)-(90) are equivalent to Eqs. (78)-(82) of the previous section apart from $\mathcal{O}(P^2)$ sources describing the deformation of the nonextremal cascading gauge theory geometry away from the near extremal D3-brane background (43). The latter is precisely the reason that the pattern of coupling of fluctuations is the same as for $P^2 = 0$.

Using Eq. (44), the most general solution of Eq. (90) is

$$z^2_{K,0} = \mathcal{C}_1 + \mathcal{C}_2 \ \ln x\,. \tag{91}$$

Regularity at the horizon and Eq. (74) imply that

$$z^2_{K,0} = 0\,. \tag{92}$$

Given Eq. (92), the most general solution to Eq. (86) takes form

$$z^2_{H,0} = \mathcal{C}_1 \left((x^2-1)\ln x - 2\right) + \mathcal{C}_2 \ (1-x^2) - \frac{1}{3K_*}\left(4 + 6K_*\beta_v\right). \tag{93}$$

Regularity at the horizon implies that $\mathcal{C}_1 = 0$, and the boundary condition (73) determines

$$\beta_v = -\frac{2}{3K_*}\,, \tag{94}$$

in agreement with [33]. Finally, since $z^0_{H,0}$ already satisfies Eq. (72), we must also set $\mathcal{C}_2$ in Eq. (93) to zero. Thus, we have

$$z^2_{H,0} = 0\,. \tag{95}$$

Remaining fluctuation equations are discussed in Appendix B.

5.3 Equations and solution in $\mathcal{O}(\mathfrak{q}^1)$ and $\mathcal{O}(P^0)$ order

Using Eqs. (84), (85), to order $\mathcal{O}(\mathfrak{q}^1)$ and $\mathcal{O}(P^0)$ we find the following set of equations

$$0 = [z^0_{H,1}]'' + \frac{1-3x^2}{x(1+x^2)}\ [z^0_{H,1}]' + \frac{4}{1+x^2}\ z^0_{H,1} + \frac{32(2(x^3-x)\kappa' + 1 + x^2)}{(1-x^4)K_*}\ z^0_{K,1}\,, \tag{96}$$

$$0 = [z^0_{f,1}]'' + \frac{1}{x}\ [z^0_{f,1}]' - \frac{8}{(1-x^2)^2}\ z^0_{f,1} + \frac{3\kappa'}{10K_*}\ [z^0_{K,1}]' + \frac{2}{(1-x^2)^2K_*}\ z^0_{K,1}\,, \tag{97}$$

$$0 =[z^0_{w,1}]'' + \frac{1}{x}\ [z^0_{w,1}]' - \frac{3}{(1-x^2)^2}\ z^0_{w,1} + \frac{\kappa'}{5K_*}\ [z^0_{K,1}]' \,, \tag{98}$$

$$0 =[z^0_{\Phi,1}]'' + \frac{1}{x}\ [z^0_{\Phi,1}]' + \frac{2\kappa'}{K_*}\ [z^0_{K,1}]' \,, \tag{99}$$

$$0 =[z^0_{K,1}]'' + \frac{1}{x}\ [z^0_{K,1}]' \,. \tag{100}$$

Notice that two sets of equations Eqs. (78)-(82) and Eqs. (96)-(100) are equivalent. With boundary conditions (72)-(74) unique solutions are

$$z^0_{H,1} = z^0_{f,1} = z^0_{w,1} = z^0_{\Phi,1} = z^0_{K,1} = 0 \,. \tag{101}$$

Again, for $z^0_{H,1}$ we reproduce the sound wave quasinormal mode in the near extremal D3-brane geometry [8,17] to order $\mathfrak{q}^1$ in the hydrodynamic approximation.

5.4 Equations and solution in $\mathcal{O}(\mathfrak{q}^1)$ and $\mathcal{O}(P^2)$ order

Using Eqs. (84), (85), (87)-(88), (92), (94), (95), (101), to order $\mathcal{O}(\mathfrak{q}^1)$ and $\mathcal{O}(P^2)$ we find the following set of equations

$$\begin{aligned}0 =&[z^2_{H,1}]'' + \frac{1-3x^2}{x(1+x^2)}\ [z^2_{H,1}]' + \frac{4}{1+x^2}\ z^2_{H,1} + \frac{32(2(x^3-x)\kappa'+1+x^2)}{(1-x^4)K_*}\ z^2_{K,1}\\ &- \frac{8\sqrt{3}}{9K_*\left(x^2+1\right)x^2}\Bigg(\left(\kappa'\right)^2\left(1-x^2\right)^2\left(\left(x^2+1\right)\ln x+3x^2+1\right)\\ &+6x\left(1-x^2\right)^2\left(\ln x+3\right)\xi'+x^2\left(3K_*\beta_\Gamma-6-2\ln x\right)\Bigg)\,,\end{aligned} \tag{102}$$

$$\begin{aligned}0 =&[z^2_{f,1}]'' + \frac{1}{x}\ [z^2_{f,1}]' - \frac{8}{(1-x^2)^2}\ z^2_{f,1} + \frac{3\kappa'}{10K_*}\ [z^2_{K,1}]' + \frac{2}{(1-x^2)^2K_*}\ z^2_{K,1}\\ &- \frac{\sqrt{3}}{60\left(x^2-1\right)^2\left(x^2+1\right)K_*x^3}\Bigg(40x^2K_*\left(1+x^2\right)\left(1-x^2\right)^2[z^2_{f,0}]'\\ &-\left(1-x^4\right)\left(40\eta'(x^2-1)-40x\kappa-7x+160x\eta\right)\Bigg)\,,\end{aligned} \tag{103}$$

$$
\begin{aligned}
0 =&[z^2_{w,1}]'' + \frac{1}{x}\ [z^2_{w,1}]' - \frac{3}{(1-x^2)^2}\ z^2_{w,1} + \frac{\kappa'}{5K_*}\ [z^2_{K,1}]' \\
&+ \frac{\sqrt{3}}{30K_* x^3(x^4-1)}\biggl(20x^2K_*(1-x^4)[z^2_{w,0}]' \\
&- (x^2+1)\left(20(x^2-1)\psi' + 30x\psi + x\right)\biggr)\,,
\end{aligned} \tag{104}
$$

$$
\begin{aligned}
0 =&[z^2_{\Phi,1}]'' + \frac{1}{x}\ [z^2_{\Phi,1}]' + \frac{2\kappa'}{K_*}\ [z^2_{K,1}]' + \frac{\sqrt{3}}{3K_* x^3(x^4-1)}\biggl(2x^2K_*(1-x^4)[z^2_{\Phi,0}]' \\
&- (x^2+1)\left(2(x^2-1)\zeta' + x\right)\biggr)\,,
\end{aligned} \tag{105}
$$

$$
0 =[z^2_{K,1}]'' + \frac{1}{x}\ [z^2_{K,1}]' - \frac{2\sqrt{3}}{9x^3(x^4-1)}\left((x^2-1)\kappa' + x\right)\left((x^2+1)\ln x + 3x^2 + 1\right)\,. \tag{106}
$$

Given Eq. (44), the most general solution to Eq. (106) is

$$
z^2_{K,1} = \mathcal{C}_1 + \mathcal{C}_2\ \ln x\,. \tag{107}
$$

Regularity at the horizon and Eq. (74) imply that

$$
z^2_{K,1} = 0\,. \tag{108}
$$

With Eq. (108), the most general solution to Eq. (102) takes the form

$$
z^2_{H,1} = \mathcal{C}_1\ \biggl((x^2-1)\ln x - 2\biggr) + \mathcal{C}_2\ (1-x^2) + \frac{2\sqrt{3}}{9K_*}\biggl(3K_*\beta_\Gamma - 2\biggr)\,. \tag{109}
$$

Regularity at the horizon implies that $\mathcal{C}_1 = 0$, and boundary condition (73) determines

$$
\beta_\Gamma = \frac{2}{3K_*}\,. \tag{110}
$$

Finally, since $z^0_{H,0}$ already satisfies Eq. (72), we must also set $\mathcal{C}_2$ in Eq. (109) to zero. Thus, we have

$$
z^2_{H,1} = 0\,. \tag{111}
$$

Remaining fluctuation equations are discussed in Appendix C.

Acknowledgments

I would like to thank Ofer Aharony for valuable discussions and comments on the manuscript. I would like to thank Ofer Aharony and Amos Yarom for collaboration on [33], which was vital for the analysis presented here. Research at Perimeter Institute is supported in part by funds from NSERC of Canada. I gratefully acknowledge support by NSERC Discovery grant. I would like to thank Aspen Center for Physics for hospitality where part of this work was done.

Appendix

A Connection coefficients for Eqs. (64)-(68)

$$A_H = 3\omega^2 c_2' c_2^2 c_3 c_1^2 \left(-c_2 c_1 q^2 c_1' - 2c_2' c_1^2 q^2 + 3\omega^2 c_2' c_2^2 \right). \tag{112}$$

$$\begin{aligned} B_H = & \omega^2 c_2 c_1 \Big(27\omega^2 c_2^2 c_1 c_2'^3 c_3 - 42 c_1^3 q^2 c_2'^3 c_3 + 9 c_2^2 c_1 q^2 c_2' c_1'^2 c_3 - 2 c_2^2 c_1^3 q^2 c_2' c_3^3 \mathcal{P} \\ & + 2 c_2^3 c_1^2 q^2 c_1' c_3^3 \mathcal{P} + 3 c_2^2 c_1^2 q^2 c_2' c_1' c_3' + 6 c_2 c_1^3 q^2 c_2'^2 c_3' - 3 c_2 c_1^2 q^2 c_2'^2 c_1' c_3 - 9\omega^2 c_2^3 c_1 c_2'^2 c_3' \\ & + 9\omega^2 c_2^3 c_2'^2 c_3 c_1' \Big). \end{aligned} \tag{113}$$

$$\begin{aligned} C_H = & c_3 \omega^2 \Big(-3\omega^2 c_3^2 c_2' c_2^3 q^2 c_1 c_1' + 9\omega^4 c_3^2 c_2'^2 c_2^4 - 2 c_3^2 q^2 c_1^4 c_2'^2 c_2^2 \mathcal{P} + 36 q^2 c_1^3 c_2'^3 c_1' c_2 \\ & - 24 q^2 c_1^4 c_2'^4 + 4 c_3^2 q^2 c_1^3 c_2' c_1' c_2^3 \mathcal{P} - 2 c_3^2 q^2 c_1^2 c_1'^2 c_2^4 \mathcal{P} + 3 c_3^2 c_2' c_1^3 q^4 c_1' c_2 + 6 c_3^2 c_2'^2 c_1^4 q^4 \\ & - 12 q^2 c_1 c_1'^3 c_2^3 c_2' - 15\omega^2 c_3^2 c_2'^2 c_1^2 q^2 c_2^2 \Big). \end{aligned} \tag{114}$$

$$\begin{aligned} D_H = & 16 c_3 q^2 c_1^2 \Big(-c_1 c_2' + c_1' c_2 \Big) \Big(-\frac{20}{3} c_2 (-c_1 c_3^2 \mathcal{P} \omega^2 c_2^2 - 6 c_2' \omega^2 c_1' c_2 + 6 c_1 (c_2')^2 \omega^2 \\ & + c_1^3 q^2 c_3^2 \mathcal{P}) f' - \frac{1}{4} c_1 c_3^2 \frac{\partial \mathcal{P}}{\partial f} (2 c_2' c_1^2 q^2 - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2) \Big). \end{aligned} \tag{115}$$

$$\begin{aligned} E_H = & 16 c_3 q^2 c_1^2 \Big(-c_1 c_2' + c_1' c_2 \Big) \Big(-10 c_2 (-c_1 c_3^2 \mathcal{P} \omega^2 c_2^2 - 6 c_2' \omega^2 c_1' c_2 + 6 c_1 c_2'^2 \omega^2 \\ & + c_1^3 q^2 c_3^2 \mathcal{P}) w' - \frac{1}{4} c_3^2 c_1 \frac{\partial \mathcal{P}}{\partial w} (2 c_2' c_1^2 q^2 - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2) \Big). \end{aligned} \tag{116}$$

$$
\begin{aligned}
F_H =&16c_3q^2c_1^2\bigg(-c_1c_2'+c_1'c_2\bigg)\bigg(-\frac{1}{4}c_2(-c_1c_3^2\mathcal{P}\omega^2c_2^2-6c_2'\omega^2c_1'c_2+6c_1c_2'^2\omega^2\\
&+c_1^3q^2c_3^2\mathcal{P})\Phi'-\frac{1}{4}c_1c_3^2\frac{\partial\mathcal{P}}{\partial\Phi}(2c_2'c_1^2q^2-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2)\bigg).
\end{aligned}
\tag{117}
$$

$$
\begin{aligned}
G_H =&16c_3q^2c_1^2\bigg(-c_1c_2'+c_1'c_2\bigg)\bigg(-\frac{1}{8P^2}c_2e^{-\Phi-4f-4w}(-c_1c_3^2\mathcal{P}\omega^2c_2^2-6c_2'\omega^2c_1'c_2\\
&+6c_1c_2'^2\omega^2+c_1^3q^2c_3^2\mathcal{P})K'-\frac{1}{4}c_3^2c_1\frac{\partial\mathcal{P}}{\partial K}(2c_2'c_1^2q^2-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2)\bigg).
\end{aligned}
\tag{118}
$$

$$
A_f =12c_2'c_3c_1^2c_2^2\bigg(-2c_2'c_1^2q^2+3c_2'c_2^2\omega^2-c_1c_2c_1'q^2\bigg). \tag{119}
$$

$$
B_f =12c_2'c_2c_1\bigg(-2c_2'c_1^2q^2+3c_2'c_2^2\omega^2-c_1c_2c_1'q^2\bigg)\bigg(3c_3c_1c_2'-c_2c_3'c_1+c_2c_1'c_3\bigg). \tag{120}
$$

$$
C_f =-c_3^3c_1^2c_2^5\omega^2\bigg(\frac{9}{40}c_2'\frac{\partial\mathcal{P}}{\partial f}+2c_2f'\mathcal{P}\bigg). \tag{121}
$$

$$
D_f =-c_3^3c_1c_2^4\omega^2\bigg(c_1c_2'-c_2c_1'\bigg)\bigg(\frac{9}{40}c_2'\frac{\partial\mathcal{P}}{\partial f}+2c_2f'\mathcal{P}\bigg). \tag{122}
$$

$$
\begin{aligned}
E_f =&\frac{9}{20}c_3^3c_2'c_1^2c_2^2\bigg(2c_2'c_1^2q^2-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\bigg)\frac{\partial^2\mathcal{P}}{\partial f^2}+4c_3^3c_2^3f'c_1^2\bigg(-6c_2'c_2^2\omega^2\\
&+5c_2'c_1^2q^2+c_1c_2c_1'q^2\bigg)\frac{\partial\mathcal{P}}{\partial f}+\frac{320}{3}c_3^3c_2^4c_1^2f'^2\bigg(-c_2^2\omega^2+c_1^2q^2\bigg)\mathcal{P}\\
&+\frac{3}{5}c_3c_2'\bigg(\frac{3}{P^2}K'^2e^{-\Phi-4f-4w}c_1^2c_2^2-20c_3^2c_2^2\omega^2+20c_3^2c_1^2q^2\bigg)\bigg(2c_2'c_1^2q^2-3c_2'c_2^2\omega^2\\
&+c_1c_2c_1'q^2\bigg).
\end{aligned}
\tag{123}
$$

$$
\begin{aligned}
F_f =& \frac{9}{20} c_1^2 c_2^2 c_3^3 c_2' \left(2 c_2' c_1^2 q^2 - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2\right) \frac{\partial^2 \mathcal{P}}{\partial f \partial w} + 18 c_2^3 c_3^3 c_1^2 c_2' w' \left(-c_2^2 \omega^2\right. \\
& \left. + c_1^2 q^2\right) \frac{\partial \mathcal{P}}{\partial f} + 4 c_1^2 c_2^3 c_3^3 f' \left(2 c_2' c_1^2 q^2 - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2\right) \frac{\partial \mathcal{P}}{\partial w} \\
& + 160 c_1^2 c_2^4 c_3^3 f' w' \left(-c_2^2 \omega^2 + c_1^2 q^2\right) \mathcal{P} + \frac{9}{5 P^2} c_2^2 c_3 c_1^2 c_2' K'^2 e^{-\Phi - 4f - 4w} \left(2 c_2' c_1^2 q^2\right. \\
& \left. - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2\right) .
\end{aligned} \tag{124}
$$

$$
\begin{aligned}
G_f =& \frac{9}{20} c_1^2 c_2^2 c_3^3 c_2' \left(2 c_2' c_1^2 q^2 - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2\right) \frac{\partial^2 \mathcal{P}}{\partial f \partial \Phi} + \frac{9}{20} c_2^3 c_3^3 c_1^2 c_2' \Phi' \left(-c_2^2 \omega^2\right. \\
& \left. + c_1^2 q^2\right) \frac{\partial \mathcal{P}}{\partial f} + 4 c_1^2 c_2^3 c_3^3 f' \left(2 c_2' c_1^2 q^2 - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2\right) \frac{\partial \mathcal{P}}{\partial \Phi} \\
& + 4 c_2^4 c_3^3 c_1^2 f' \Phi' \left(-c_2^2 \omega^2 + c_1^2 q^2\right) \mathcal{P} + \frac{9}{20 P^2} c_2^2 c_3 c_1^2 c_2' K'^2 e^{-\Phi - 4f - 4w} \left(2 c_2' c_1^2 q^2\right. \\
& \left. - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2\right) .
\end{aligned} \tag{125}
$$

$$
H_f = - \frac{9}{10 P^2} c_1^2 c_2^2 c_3 c_2' K' e^{-\Phi - 4f - 4w} \left(2 c_2' c_1^2 q^2 - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2\right) . \tag{126}
$$

$$
\begin{aligned}
I_f =& \frac{9}{20} c_3^3 c_1^2 c_2^2 c_2' \left(2 c_2' c_1^2 q^2 - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2\right) \frac{\partial^2 \mathcal{P}}{\partial f \partial K} \\
& + \frac{9}{40 P^2} c_2^3 c_3^3 c_1^2 c_2' K' e^{-\Phi - 4f - 4w} \left(-c_2^2 \omega^2 + c_1^2 q^2\right) \frac{\partial \mathcal{P}}{\partial f} + 4 c_2^3 c_3^3 c_1^2 f' \left(2 c_2' c_1^2 q^2\right. \\
& \left. - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2\right) \frac{\partial \mathcal{P}}{\partial K} + \frac{2}{P^2} c_2^4 c_3^3 c_1^2 f' K' e^{-\Phi - 4f - 4w} \left(-c_2^2 \omega^2 + c_1^2 q^2\right) \mathcal{P} .
\end{aligned} \tag{127}
$$

$$
A_w = 12 c_2' c_3 c_1^2 c_2^2 \left(-2 c_2' c_1^2 q^2 + 3 c_2' c_2^2 \omega^2 - c_1 c_2 c_1' q^2\right) . \tag{128}
$$

$$
B_w = 12 c_2' c_2 c_1 \left(-2 c_2' c_1^2 q^2 + 3 c_2' c_2^2 \omega^2 - c_1 c_2 c_1' q^2\right) \left(3 c_3 c_1 c_2' - c_2 c_3' c_1 + c_2 c_1' c_3\right) . \tag{129}
$$

$$
C_w = - c_3^3 c_1^2 c_2^5 \omega^2 \left(2 c_2 w' \mathcal{P} + \frac{3}{20} c_2' \frac{\partial \mathcal{P}}{\partial w}\right) . \tag{130}
$$

$$D_w = - c_3^3 c_1 c_2^4 \omega^2 \left(c_1 c_2' - c_2 c_1' \right) \left(2 c_2 w' \mathcal{P} + \frac{3}{20} c_2' \frac{\partial \mathcal{P}}{\partial w} \right) . \tag{131}$$

$$\begin{aligned} E_w =& \frac{3}{10} c_1^2 c_2^2 c_3^3 c_2' \left(2 c_2' c_1^2 q^2 - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2 \right) \frac{\partial^2 \mathcal{P}}{\partial f \partial w} + 4 c_3^3 c_1^2 c_2^3 w' \Big(2 c_2' c_1^2 q^2 \\ & - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2 \Big) \frac{\partial \mathcal{P}}{\partial f} + 8 c_3^3 c_1^2 c_2^3 c_2' f' \left(- c_2^2 \omega^2 + c_1^2 q^2 \right) \frac{\partial \mathcal{P}}{\partial w} \\ & + \frac{320}{3} c_1^2 c_2^4 c_3^3 f' w' \left(- c_2^2 \omega^2 + c_1^2 q^2 \right) \mathcal{P} + \frac{6}{5 P^2} c_2^2 c_3 c_1^2 c_2' K'^2 e^{-\Phi - 4f - 4w} \Big(2 c_2' c_1^2 q^2 \\ & - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2 \Big) . \end{aligned} \tag{132}$$

$$\begin{aligned} F_w =& \frac{3}{10} c_3^3 c_2' c_1^2 c_2^2 \left(2 c_2' c_1^2 q^2 - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2 \right) \frac{\partial^2 \mathcal{P}}{\partial w^2} + 4 c_3^3 c_2^3 c_1^2 w' \Big(- 6 c_2' c_2^2 \omega^2 \\ & + 5 c_2' c_1^2 q^2 + c_1 c_2 c_1' q^2 \Big) \frac{\partial \mathcal{P}}{\partial w} + 160 c_3^3 c_2^4 c_1^2 w'^2 \left(- c_2^2 \omega^2 + c_1^2 q^2 \right) \mathcal{P} \\ & + \frac{6}{5} c_3 c_2' \left(\frac{1}{P^2} K'^2 e^{-\Phi - 4f - 4w} c_1^2 c_2^2 - 10 c_3^2 c_2^2 \omega^2 + 10 c_3^2 c_1^2 q^2 \right) \Big(2 c_2' c_1^2 q^2 - 3 c_2' c_2^2 \omega^2 \\ & + c_1 c_2 c_1' q^2 \Big) . \end{aligned} \tag{133}$$

$$\begin{aligned} G_w =& \frac{3}{10} c_3^3 c_2' c_1^2 c_2^2 \left(2 c_2' c_1^2 q^2 - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2 \right) \frac{\partial^2 \mathcal{P}}{\partial w \partial \Phi} + \frac{3}{10} c_1^2 c_2^3 c_3^3 c_2' \Phi' \Big(- c_2^2 \omega^2 \\ & + c_1^2 q^2 \Big) \frac{\partial \mathcal{P}}{\partial w} + 4 c_1^2 c_2^3 c_3^3 w' \left(2 c_2' c_1^2 q^2 - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2 \right) \frac{\partial \mathcal{P}}{\partial \Phi} \\ & + 4 c_1^2 c_2^4 c_3^3 w' \Phi' \left(- c_2^2 \omega^2 + c_1^2 q^2 \right) \mathcal{P} + \frac{3}{10 P^2} c_2^2 c_3 c_1^2 c_2' K'^2 e^{-\Phi - 4f - 4w} \Big(2 c_2' c_1^2 q^2 \\ & - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2 \Big) . \end{aligned} \tag{134}$$

$$H_w = - \frac{3}{5 P^2} c_1^2 c_2^2 c_3 c_2' K' e^{-\Phi - 4f - 4w} \left(2 c_2' c_1^2 q^2 - 3 c_2' c_2^2 \omega^2 + c_1 c_2 c_1' q^2 \right) . \tag{135}$$

$$
\begin{aligned}
I_w =&\frac{3}{10}c_2^2c_3^3c_1^2c_2'\left(2c_2'c_1^2q^2-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\right)\frac{\partial^2\mathcal{P}}{\partial w\partial K}\\
&+\frac{3}{20P^2}c_2^3c_3^3c_1^2c_2'K'e^{-\Phi-4f-4w}\left(-c_2^2\omega^2+c_1^2q^2\right)\frac{\partial\mathcal{P}}{\partial w}+4c_2^3c_3^3c_1^2w'\Big(2c_2'c_1^2q^2\\
&-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\Big)\frac{\partial\mathcal{P}}{\partial K}+\frac{2}{P^2}c_2^4c_3^3c_1^2w'K'e^{-\Phi-4f-4w}\Big(-c_2^2\omega^2+c_1^2q^2\Big)\mathcal{P}\,.
\end{aligned}
\tag{136}
$$

$$
A_\Phi =12c_2'c_3c_1^2c_2^2\left(-2c_2'c_1^2q^2+3c_2'c_2^2\omega^2-c_1c_2c_1'q^2\right). \tag{137}
$$

$$
B_\Phi =12c_2'c_2c_1\left(-2c_2'c_1^2q^2+3c_2'c_2^2\omega^2-c_1c_2c_1'q^2\right)\left(3c_3c_1c_2'-c_2c_3'c_1+c_2c_1'c_3\right). \tag{138}
$$

$$
C_\Phi =-c_3^3c_1^2c_2^5\omega^2\left(6c_2'\frac{\partial\mathcal{P}}{\partial\Phi}+2c_2\Phi'\mathcal{P}\right). \tag{139}
$$

$$
D_\Phi =-c_3^3c_1c_2^4\omega^2\left(c_1c_2'-c_2c_1'\right)\left(6c_2'\frac{\partial\mathcal{P}}{\partial\Phi}+2c_2\Phi'\mathcal{P}\right). \tag{140}
$$

$$
\begin{aligned}
E_\Phi =&12c_1^2c_2^2c_3^3c_2'\left(2c_2'c_1^2q^2-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\right)\frac{\partial^2\mathcal{P}}{\partial f\partial\Phi}+4c_3^3c_2^3c_1^2\Phi'\Big(2c_2'c_1^2q^2\\
&-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\Big)\frac{\partial\mathcal{P}}{\partial f}+320c_3^3c_2^3c_1^2c_2'f'\left(-c_2^2\omega^2+c_1^2q^2\right)\frac{\partial\mathcal{P}}{\partial\Phi}\\
&+\frac{320}{3}c_2^4c_3^3c_1^2f'\Phi'\left(-c_2^2\omega^2+c_1^2q^2\right)\mathcal{P}+\frac{12}{P^2}c_2^2c_3c_1^2c_2'K'^2e^{-\Phi-4f-4w}\Big(2c_2'c_1^2q^2\\
&-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\Big).
\end{aligned}
\tag{141}
$$

$$
\begin{aligned}
F_\Phi =&12c_3^3c_2'c_1^2c_2^2\left(2c_2'c_1^2q^2-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\right)\frac{\partial^2\mathcal{P}}{\partial w\partial\Phi}+4c_3^3c_1^2c_2^3\Phi'\Big(2c_2'c_1^2q^2\\
&-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\Big)\frac{\partial\mathcal{P}}{\partial w}+480c_3^3c_1^2c_2^3c_2'w'\left(-c_2^2\omega^2+c_1^2q^2\right)\frac{\partial\mathcal{P}}{\partial\Phi}\\
&+160c_1^2c_2^4c_3^3w'\Phi'\left(-c_2^2\omega^2+c_1^2q^2\right)\mathcal{P}+\frac{12}{P^2}c_2^2c_3c_1^2c_2'K'^2e^{-\Phi-4f-4w}\Big(2c_2'c_1^2q^2\\
&-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\Big).
\end{aligned}
\tag{142}
$$

$$
\begin{aligned}
G_\Phi =& 12c_3^3c_2'c_1^2c_2^2\left(2c_2'c_1^2q^2-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\right)\frac{\partial^2\mathcal{P}}{\partial\Phi^2}+4c_3^3c_2^3\Phi'c_1^2\Big(-6c_2'c_2^2\omega^2\\
&+5c_2'c_1^2q^2+c_1c_2c_1'q^2\Big)\frac{\partial\mathcal{P}}{\partial\Phi}+4c_3^3c_2^4c_1^2\Phi'^2\left(-c_2^2\omega^2+c_1^2q^2\right)\mathcal{P}\\
&+3c_3c_2'\left(\frac{1}{P^2}K'^2e^{-\Phi-4f-4w}c_1^2c_2^2-4c_3^2c_2^2\omega^2+4c_3^2c_1^2q^2\right)\Big(2c_2'c_1^2q^2-3c_2'c_2^2\omega^2\\
&+c_1c_2c_1'q^2\Big)\,.
\end{aligned}
\tag{143}
$$

$$
H_\Phi =-\frac{6}{P^2}c_1^2c_2^2c_3c_2'K'e^{-\Phi-4f-4w}\left(2c_2'c_1^2q^2-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\right)\,.
\tag{144}
$$

$$
\begin{aligned}
I_\Phi =& 12c_2^2c_3^3c_1^2c_2'\left(2c_2'c_1^2q^2-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\right)\frac{\partial^2\mathcal{P}}{\partial\Phi\partial K}\\
&+\frac{6}{P^2}c_2^3c_3^3c_1^2c_2'K'e^{-\Phi-4f-4w}\left(-c_2^2\omega^2+c_1^2q^2\right)\frac{\partial\mathcal{P}}{\partial\Phi}+4c_2^3c_3^3c_1^2\Phi'\Big(2c_2'c_1^2q^2\\
&-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\Big)\frac{\partial\mathcal{P}}{\partial K}+\frac{2}{P^2}c_2^4c_3^3c_1^2\Phi'K'e^{-\Phi-4f-4w}\left(-c_2^2\omega^2+c_1^2q^2\right)\mathcal{P}\,.
\end{aligned}
\tag{145}
$$

$$
A_K =12c_2'c_3c_1^2c_2^2\left(-2c_2'c_1^2q^2+3c_2'c_2^2\omega^2-c_1c_2c_1'q^2\right)\,.
\tag{146}
$$

$$
\begin{aligned}
B_K =& 12c_2'c_2c_1\left(-2c_2'c_1^2q^2+3c_2'c_2^2\omega^2-c_1c_2c_1'q^2\right)\left(3c_3c_1c_2'-c_2c_3'c_1+c_2c_1'c_3\right)\\
&+12c_3c_2'c_1^2c_2^2\left(4w'+\Phi'+4f'\right)\left(2c_2'c_1^2q^2-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\right)\,.
\end{aligned}
\tag{147}
$$

$$
C_K =-c_3^3c_1^2c_2^5\omega^2\left(12c_2'P^2\frac{\partial\mathcal{P}}{\partial K}e^{\Phi+4f+4w}+2c_2K'\mathcal{P}\right)\,.
\tag{148}
$$

$$
D_K =-c_3^3c_1c_2^4\omega^2\left(c_1c_2'-c_2c_1'\right)\left(12c_2'P^2\frac{\partial\mathcal{P}}{\partial K}e^{\Phi+4f+4w}+2c_2K'\mathcal{P}\right)\,.
\tag{149}
$$

$$
E_K =48c_3c_2'K'c_1^2c_2^2\left(2c_2'c_1^2q^2-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\right)\,.
\tag{150}
$$

$$
\begin{aligned}
F_K =& 24c_3^3c_1^2c_2^2c_2'P^2e^{\Phi+4f+4w}\left(2c_2'c_1^2q^2 - 3c_2'c_2^2\omega^2 + c_1c_2c_1'q^2\right)\frac{\partial^2\mathcal{P}}{\partial f\partial K} \\
&+ 4c_3^3c_1^2c_2^3K'\left(2c_2'c_1^2q^2 - 3c_2'c_2^2\omega^2 + c_1c_2c_1'q^2\right)\frac{\partial\mathcal{P}}{\partial f} \\
&+ 32c_3^3c_1^2c_2^2c_2'P^2e^{\Phi+4f+4w}\left(20c_1^2c_2q^2f' - 20c_2^3\omega^2f' - 9c_2'c_2^2\omega^2 + 3c_1c_2c_1'q^2\right. \\
&\left.+ 6c_2'c_1^2q^2\right)\frac{\partial\mathcal{P}}{\partial K} + \frac{320}{3}c_3^3c_1^2c_2^4K'f'\left(-c_2^2\omega^2 + c_1^2q^2\right)\mathcal{P}\,.
\end{aligned}
\tag{151}
$$

$$
G_K = 48c_3c_2'K'c_1^2c_2^2\left(2c_2'c_1^2q^2 - 3c_2'c_2^2\omega^2 + c_1c_2c_1'q^2\right). \tag{152}
$$

$$
\begin{aligned}
H_K =& 24c_3^3c_2^2c_1^2c_2'P^2e^{\Phi+4f+4w}\left(2c_2'c_1^2q^2 - 3c_2'c_2^2\omega^2 + c_1c_2c_1'q^2\right)\frac{\partial^2\mathcal{P}}{\partial w\partial K} \\
&+ 4c_3^3c_2^3c_1^2K'\left(2c_2'c_1^2q^2 - 3c_2'c_2^2\omega^2 + c_1c_2c_1'q^2\right)\frac{\partial\mathcal{P}}{\partial w} \\
&+ 96c_3^3c_2^2c_1^2c_2'P^2e^{\Phi+4f+4w}\left(10c_1^2c_2q^2w' + 2c_2'c_1^2q^2 - 10c_2^3\omega^2w' - 3c_2'c_2^2\omega^2\right. \\
&\left.+ c_1c_2c_1'q^2\right)\frac{\partial\mathcal{P}}{\partial K} + 160c_3^3c_2^4c_1^2K'w'\left(-c_2^2\omega^2 + c_1^2q^2\right)\mathcal{P}\,.
\end{aligned}
\tag{153}
$$

$$
I_K = 12c_3c_2'K'c_1^2c_2^2\left(2c_2'c_1^2q^2 - 3c_2'c_2^2\omega^2 + c_1c_2c_1'q^2\right). \tag{154}
$$

$$
\begin{aligned}
J_K =& 24c_3^3c_2^2c_1^2c_2'P^2e^{\Phi+4f+4w}\left(2c_2'c_1^2q^2 - 3c_2'c_2^2\omega^2 + c_1c_2c_1'q^2\right)\frac{\partial^2\mathcal{P}}{\partial\Phi\partial K} \\
&+ 4c_3^3c_2^3c_1^2K'\left(2c_2'c_1^2q^2 - 3c_2'c_2^2\omega^2 + c_1c_2c_1'q^2\right)\frac{\partial\mathcal{P}}{\partial\Phi} \\
&+ 24c_3^3c_2^2c_1^2c_2'P^2e^{\Phi+4f+4w}\left(c_1c_2c_1'q^2 - c_2^3\omega^2\Phi' + c_1^2c_2q^2\Phi' + 2c_2'c_1^2q^2\right. \\
&\left.- 3c_2'c_2^2\omega^2\right)\frac{\partial\mathcal{P}}{\partial K} + 4c_3^3c_2^4c_1^2K'\Phi'\left(-c_2^2\omega^2 + c_1^2q^2\right)\mathcal{P}\,.
\end{aligned}
\tag{155}
$$

$$
\begin{aligned}
K_K =&24c_3^3c_2^2c_1^2c_2'P^2e^{\Phi+4f+4w}\biggl(2c_2'c_1^2q^2-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\biggr)\frac{\partial^2\mathcal{P}}{\partial K^2}\\
&+4c_3^3c_2^3K'c_1^2\biggl(-6c_2'c_2^2\omega^2+5c_2'c_1^2q^2+c_1c_2c_1'q^2\biggr)\frac{\partial\mathcal{P}}{\partial K}\\
&+\frac{2}{P^2}c_3^3c_2^4c_1^2K'^2e^{-\Phi-4f-4w}\biggl(-c_2^2\omega^2+c_1^2q^2\biggr)\mathcal{P}\\
&+12c_2'c_3^3\biggl(-c_2^2\omega^2+c_1^2q^2\biggr)\biggl(2c_2'c_1^2q^2-3c_2'c_2^2\omega^2+c_1c_2c_1'q^2\biggr).
\end{aligned}
\tag{156}
$$

B Matter fluctuations at order $\mathcal{O}(\mathfrak{q}^0)$ and $\mathcal{O}(P^2)$

Unique solution to Eq. (89) subject to regularity at the horizon and the boundary condition (74) is

$$
z_{\Phi,0}^2=\frac{1-x^2}{12K_*x^2}\ \ln(1-x^2)\,.
\tag{157}
$$

Eqs. (87), (88) are second order linear non-homogeneous ordinary differential equations which general homogeneous solution can be found analytically. Thus it is straightforward to write down a formal solution to (87), (88) in quadratures. We do not need these explicit expressions for the computation of the hydrodynamic properties of the high-temperature cascading gauge theory plasma, so we only argue here that regularity at the horizon and the Dirichlet condition at the boundary determine two integration constants. We verified numerically that these are the two *independent* integration constants. The latter implies that $z_{f,0}^2$, $z_{w,0}^2$ are uniquely determined.

Consider first Eq. (87). Its general solution near the horizon $x\to 0_+$ and the boundary $(1-x^2)\equiv y\to 0_+$ takes form

$$
z_{f,0}^2=\mathcal{C}_1\ (2+\ln x)+\mathcal{C}_2+\mathcal{O}(x^2\ln x)\,,
\tag{158}
$$

$$
z_{f,0}^2=\frac{1}{y}\ \left(-2\hat{\mathcal{C}}_2-4\hat{\mathcal{C}}_1-\frac{3}{40K_*}\right)+\left(\hat{\mathcal{C}}_2+2\hat{\mathcal{C}}_1+\frac{3}{80K_*}\right)-\frac{1}{160K_*}\ y+\mathcal{O}(y^2\ln y)\,,
\tag{159}
$$

where $\hat{\mathcal{C}}_1=\hat{\mathcal{C}}_1(\mathcal{C}_1,\mathcal{C}_2)$ and $\hat{\mathcal{C}}_2=\hat{\mathcal{C}}_2(\mathcal{C}_1,\mathcal{C}_2)$. Regularity at the horizon implies that $\mathcal{C}_1=0$; Dirichlet condition at the boundary further constraints $\mathcal{C}_2$:

$$
0=\hat{\mathcal{C}}_2+2\hat{\mathcal{C}}_1+\frac{3}{80K_*}=\hat{\mathcal{C}}_2\,(0,\mathcal{C}_2)+2\hat{\mathcal{C}}_1\,(0,\mathcal{C}_2)+\frac{3}{80K_*}\,.
\tag{160}
$$

We verified numerically that Eq. (160) indeed has a solution.

Similarly, the general solution near the horizon $x \to 0_+$ and the boundary $(1-x^2) \equiv y \to 0_+$ of Eq. (88) takes form

$$z_{w,0}^2 = \mathcal{C}_1 \ \ln x + \mathcal{C}_2 + \mathcal{O}(x^2 \ln x)\,, \tag{161}$$

$$K_* \ z_{w,0}^2 = \frac{1}{y^{1/2}} \ \hat{\mathcal{C}}_1 - \frac{1}{4} y^{1/2} \ \hat{\mathcal{C}}_1 - \frac{1}{90} y + y^{3/2} \left(\hat{\mathcal{C}}_2 - \frac{1}{32} \ \hat{\mathcal{C}}_1 \ \ln y \right) + \mathcal{O}\left(y^2\right) \ , \tag{162}$$

where we used the power series solution for ψ (see Eqs. (45), (46)) near the horizon

$$\psi = \psi_{horizon} + x^2 \left(\frac{3}{4} \ \psi_{horizon} + \frac{1}{40} \right) + \mathcal{O}(x^4)\,, \tag{163}$$

and at the boundary

$$\psi = -\frac{1}{30} \ y + \hat{\psi} \ y^{3/2} - \frac{2}{75} \ y^2 + \mathcal{O}\left(y^{5/2}\right) \ . \tag{164}$$

Parameters $\psi_{horizon}$, $\hat{\psi} = \hat{\psi}(\phi_{horizon})$ are uniquely determined from the second boundary condition in Eq. (46). In Eq. (162) $\hat{\mathcal{C}}_1 = \hat{\mathcal{C}}_1(\mathcal{C}_1, \mathcal{C}_2)$ and $\hat{\mathcal{C}}_2 = \hat{\mathcal{C}}_2(\mathcal{C}_1, \mathcal{C}_2)$. Regularity at the horizon implies that $\mathcal{C}_1 = 0$; Dirichlet condition at the boundary further constraints $\mathcal{C}_2$:

$$0 = \hat{\mathcal{C}}_1 = \hat{\mathcal{C}}_1\left(0, \mathcal{C}_2\right) \ . \tag{165}$$

We verified numerically that Eq. (165) indeed has a solution.

C Matter fluctuations at order $\mathcal{O}(\mathfrak{q}^1)$ and $\mathcal{O}(P^2)$

Unique solution to Eq. (105) subject to regularity at the horizon and the boundary condition (74) is

$$z_{\Phi,1}^2 = -\frac{\sqrt{3}}{36 K_* x^2} \left(2x^2 \left(\ln x - 2\right) \ln(1-x^2) + x^2 \mathrm{dilog}(\mathrm{x}^2) + 4 \ln(1-\mathrm{x}^2) \right) . \tag{166}$$

As in Appendix B, here we only argue that $z_{f,1}^2, z_{w,1}^2$ solutions to Eqs. (103) and (104) with appropriate boundary conditions exist, and are unique.

Consider first Eq. (103). Its general solution near the horizon $x \to 0_+$ and the boundary $(1-x^2) \equiv y \to 0_+$ takes form

$$z_{f,1}^2 = \mathcal{C}_1 \ (2 + \ln x) + \mathcal{C}_2 + \mathcal{O}(x^2 \ln x)\,, \tag{167}$$

$$z^2_{f,1} = \frac{1}{y}\ \left(-2\hat{\mathcal{C}}_2 - 4\hat{\mathcal{C}}_1\right) + \left(\hat{\mathcal{C}}_2 + 2\hat{\mathcal{C}}_1\right) + \frac{\sqrt{3}}{160K_*}\ y + \mathcal{O}(y^2 \ln y)\,, \tag{168}$$

where we used power series solution at the horizon (boundary) for $z^2_{f,0}$. Regularity at the horizon implies that $\mathcal{C}_1 = 0$; Dirichlet condition at the boundary further constraints $\mathcal{C}_2$:

$$0 = \hat{\mathcal{C}}_2 + 2\hat{\mathcal{C}}_1 = \hat{\mathcal{C}}_2\left(0, \mathcal{C}_2\right) + 2\hat{\mathcal{C}}_1\left(0, \mathcal{C}_2\right)\ . \tag{169}$$

We verified numerically that Eq. (169) indeed has a solution.

Finally, the general solution near the horizon $x \to 0_+$ and the boundary $(1 - x^2) \equiv y \to 0_+$ of Eq. (104) takes form

$$z^2_{w,1} = \mathcal{C}_1\ \ln x + \mathcal{C}_2 + \mathcal{O}(x^2 \ln x)\,, \tag{170}$$

$$K_*\ \times\ z^2_{w,1} = \frac{1}{y^{1/2}}\ \hat{\mathcal{C}}_1 - \frac{1}{4} y^{1/2}\ \hat{\mathcal{C}}_1 + \frac{\sqrt{3}}{90} y + y^{3/2}\left(\hat{\mathcal{C}}_2 - \frac{1}{32}\ \hat{\mathcal{C}}_1\ \ln y\right) + \mathcal{O}\left(y^2\right)\ , \tag{171}$$

where we used the power series solutions for ψ and $z^2_{w,0}$ near the horizon (boundary). Regularity at the horizon implies that $\mathcal{C}_1 = 0$; Dirichlet condition at the boundary further constraints $\mathcal{C}_2$:

$$0 = \hat{\mathcal{C}}_1 = \hat{\mathcal{C}}_1\left(0, \mathcal{C}_2\right)\ . \tag{172}$$

We verified numerically that Eq. (172) indeed has a solution.

References

[1] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].

[2] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, Phys. Rept. **323**, 183 (2000) [arXiv:hep-th/9905111].

[3] D. T. Son and A. O. Starinets, "Minkowski-space correlators in AdS/CFT correspondence: Recipe and applications," JHEP **0209**, 042 (2002) [arXiv:hep-th/0205051].

[4] C. P. Herzog and D. T. Son, "Schwinger-Keldysh propagators from AdS/CFT correspondence," JHEP **0303**, 046 (2003) [arXiv:hep-th/0212072].

[5] G. Policastro, D. T. Son and A. O. Starinets, "The shear viscosity of strongly coupled $N = 4$ supersymmetric Yang-Mills plasma," Phys. Rev. Lett. **87**, 081601 (2001) [arXiv:hep-th/0104066].

[6] G. Policastro, D. T. Son and A. O. Starinets, "From AdS/CFT correspondence to hydrodynamics," JHEP **0209**, 043 (2002) [arXiv:hep-th/0205052].

[7] C. P. Herzog, "The hydrodynamics of M-theory," JHEP **0212**, 026 (2002) [arXiv:hep-th/0210126].

[8] G. Policastro, D. T. Son and A. O. Starinets, "From AdS/CFT correspondence to hydrodynamics. II: Sound waves," JHEP **0212**, 054 (2002) [arXiv:hep-th/0210220].

[9] C. P. Herzog, "The sound of M-theory," Phys. Rev. D **68**, 024013 (2003) [arXiv:hep-th/0302086].

[10] P. Kovtun, D. T. Son and A. O. Starinets, "Holography and hydrodynamics: Diffusion on stretched horizons," JHEP **0310**, 064 (2003) [arXiv:hep-th/0309213].

[11] A. Buchel and J. T. Liu, "Universality of the shear viscosity in supergravity," Phys. Rev. Lett. **93**, 090602 (2004) [arXiv:hep-th/0311175].

[12] P. Kovtun, D. T. Son and A. O. Starinets, "Viscosity in strongly interacting quantum field theories from black hole physics," Phys. Rev. Lett. **94**, 111601 (2005) [arXiv:hep-th/0405231].

[13] A. Buchel, "N = 2* hydrodynamics," Nucl. Phys. B **708**, 451 (2005) [arXiv:hep-th/0406200].

[14] A. Buchel, J. T. Liu and A. O. Starinets, "Coupling constant dependence of the shear viscosity in N = 4 supersymmetric Yang-Mills theory," Nucl. Phys. B **707**, 56 (2005) [arXiv:hep-th/0406264].

[15] A. Buchel, "On universality of stress-energy tensor correlation functions in supergravity," Phys. Lett. B **609**, 392 (2005) [arXiv:hep-th/0408095].

[16] P. K. Kovtun and A. O. Starinets, "Quasinormal modes and holography," arXiv:hep-th/0506184.

[17] P. Benincasa, A. Buchel and A. O. Starinets, "Sound waves in strongly coupled non-conformal gauge theory plasma," arXiv:hep-th/0507026.

[18] D. Teaney, "Effect of shear viscosity on spectra, elliptic flow, and Hanbury Brown-Twiss radii," Phys. Rev. C **68**, 034913 (2003).

[19] E. Shuryak, "Why does the quark gluon plasma at RHIC behave as a nearly ideal fluid?," Prog. Part. Nucl. Phys. **53**, 273 (2004) [arXiv:hep-ph/0312227].

[20] D. Molnar and M. Gyulassy, "Saturation of elliptic flow at RHIC: Results from the covariant elastic parton cascade model MPC," Nucl. Phys. A **697**, 495 (2002) [Erratum-ibid. A **703**, 893 (2002)] [arXiv:nucl-th/0104073].

[21] R. Donagi and E. Witten, "Supersymmetric Yang-Mills Theory And Integrable Systems," Nucl. Phys. B **460**, 299 (1996) [arXiv:hep-th/9510101].

[22] K. Pilch and N. P. Warner, "N = 2 supersymmetric RG flows and the IIB dilaton," Nucl. Phys. B **594**, 209 (2001) [arXiv:hep-th/0004063].

[23] A. Buchel, A. W. Peet and J. Polchinski, "Gauge dual and noncommutative extension of an N = 2 supergravity solution," Phys. Rev. D **63**, 044009 (2001) [arXiv:hep-th/0008076].

[24] N. J. Evans, C. V. Johnson and M. Petrini, "The enhancon and N = 2 gauge theory/gravity RG flows," JHEP **0010**, 022 (2000) [arXiv:hep-th/0008081].

[25] A. Buchel and J. T. Liu, "Thermodynamics of the N = 2* flow," JHEP **0311**, 031 (2003) [arXiv:hep-th/0305064].

[26] I. R. Klebanov and N. A. Nekrasov, "Gravity duals of fractional branes and logarithmic RG flow," Nucl. Phys. B **574**, 263 (2000) [arXiv:hep-th/9911096].

[27] I. R. Klebanov and A. A. Tseytlin, "Gravity duals of supersymmetric SU(N) x SU(N+M) gauge theories," Nucl. Phys. B **578**, 123 (2000) [arXiv:hep-th/0002159].

[28] I. R. Klebanov and M. J. Strassler, "Supergravity and a confining gauge theory: Duality cascades and chiSB-resolution of naked singularities," JHEP **0008**, 052 (2000) [arXiv:hep-th/0007191].

[29] M. J. Strassler, "The duality cascade," arXiv:hep-th/0505153.

[30] A. Buchel, "Finite temperature resolution of the Klebanov-Tseytlin singularity," Nucl. Phys. B **600**, 219 (2001) [arXiv:hep-th/0011146].

[31] A. Buchel, C. P. Herzog, I. R. Klebanov, L. A. Pando Zayas and A. A. Tseytlin, "Non-extremal gravity duals for fractional D3-branes on the conifold," JHEP **0104**, 033 (2001) [arXiv:hep-th/0102105].

[32] S. S. Gubser, C. P. Herzog, I. R. Klebanov and A. A. Tseytlin, "Restoration of chiral symmetry: A supergravity perspective," JHEP **0105**, 028 (2001) [arXiv:hep-th/0102172].

[33] O. Aharony, A. Buchel and A. Yarom, "Holographic renormalization of cascading gauge theories," arXiv:hep-th/0506002.

[34] A. Parnachev and A. Starinets, "The silence of the little strings," arXiv:hep-th/0506144.

[35] A. Buchel, "A holographic perspective on Gubser-Mitra conjecture," arXiv:hep-th/0507275.

[36] A. Hosoya, M. a. Sakagami and M. Takao, "Nonequilibrium Thermodynamics In Field Theory: Transport Coefficients," Annals Phys. **154**, 229 (1984).

[37] R. Horsley and W. Schoenmaker, "Quantum Field Theories Out Of Thermal Equilibrium. 1. General Considerations," Nucl. Phys. B **280**, 716 (1987).

[38] S. Jeon and L. G. Yaffe, "From Quantum Field Theory to Hydrodynamics: Transport Coefficients and Effective Kinetic Theory," Phys. Rev. D **53**, 5799 (1996) [arXiv:hep-ph/9512263].

[39] I. R. Klebanov and E. Witten, "Superconformal field theory on threebranes at a Calabi-Yau singularity," Nucl. Phys. B **536**, 199 (1998) [arXiv:hep-th/9807080].